Periodic diffraction patterns for selected quasicrystals, Janusz Wolny[a], Paweł Buczek[a] and Lorenzo Sadun[b], [a]*Faculty of Physics and Nuclear Techniques, AGH-UTS, al. Mickiewicza 30, 30-059 Krakow, Poland,* [b]*Department of Mathematics, University of Texas, 1 University Station C1200, Austin, TX 78703, USA.* E-mail: wolny@novell.ftj.agh.edu.pl

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In this paper we consider a one-parameter family of 1D quasicrystals. Specifically, our model is a fixed (Fibonacci) sequence of two types of "atoms" with a varying amount of space around each type of atom, under assumption that the overall density is fixed. The control parameter κ is the ratio of the two allowed distances between nearest neighbors. In all cases, the diffraction pattern is discrete, and the locations of the Bragg peaks are independent of κ. However, the *intensities* of the peaks are κ-dependent. When κ is rational, the intensities form a periodic pattern, while when κ is irrational, the diffraction pattern is aperiodic. We compute this diffraction pattern in two independent but equivalent ways: a) by recovering periodicity going to higher dimension (the "cut and project method"); b) using the concept of the reference lattice (proposed in [1]), taking advantage of physical space properties of the set only. As a result we get that the amplitude of each peak is a continuous function of κ. In fact, it is infinitely differentiable. As κ is varied, there is no phase transition between commensurate and incommensurate diffraction patterns; the evolution is smooth. As such, with measurement apparatus of fixed accuracy, it is impossible to determine whether a given pattern is precisely periodic. These results are compared to the ergodic theory of tiling spaces. It is known that the Bragg peaks of a tiling *T* occur at eigenvalues of the generator of translations on the hull of *T* (i.e., the space of all tilings in the same local isomorphism class as *T*) [2]. It is also known [3] that the hulls of modified Fibonacci chains with the same average spacing are topologically conjugate, hence that their generators of translations have the same spectral decomposition. The question of when and how such a modification affects the dynamical spectrum was addressed for one dimensional patterns in [4]. (It should be noted that for a substitution tiling whose substitution matrix has two of more eigenvalues greater than 1, a generic change in tile length will destroy the Bragg peaks altogether, in sharp contrast to the behavior of modified Fibonacci chains, other Pisot substitutions, and other Sturmian sequences.) Ergodic theory says nothing, however, about the intensities of the Bragg peaks. Although the spectrum of the generator of translations is complicated, for special values of the control parameter some of the peaks may have intensity zero, resulting in a simpler diffraction pattern. The calculations in this paper demonstrate that this does in fact happen.

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